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Chiral symmetry breaking in three-dimensional QED with Abelian Higgs model

Hua Jiang, Guo-Zhu Liu and Geng Cheng

Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

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Abstract
We study the dynamical chiral symmetry breaking in (2+1)-dimensional QED in the presence of an Abelian Higgs model (Ginzburg–Landau model) at the leading order of $1/N$. In the gauge symmetry broken phase, the gauge boson becomes massive via the Anderson–Higgs mechanism. The Dyson–Schwinger equation for fermion self-energy depends on two parameters: the gauge boson mass m_A and the Higgs boson mass m_h. It is found that, in the region of large ratio $r = m_h/m_A$, m_A and m_h reduces the critical fermion number N_c, below which the massless fermion acquires a dynamical mass.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Quantum electrodynamics in 2+1 dimensions (QED_3) has been investigated extensively for more than 20 years. The motivation is twofold. As a relatively simple gauge theory, QED_3 of massless fermion exhibits dynamical chiral symmetry breaking (DCSB) [1–10], asymptotic freedom [11] and fermion confinement [12]. Therefore, it can serve as a toy model to study these fundamental phenomena in more complicated gauge theories like QCD. On the other hand, QED_3 and its non-relativistic variants have been used to model the physics of a series of planar condensed matter systems, including high-temperature superconductors [13–24], quantum Hall systems [25–27], quantum Heisenberg antiferromagnets [15, 19], organic compounds [28] and graphene [29, 30]. Recently, it becomes more transparent that some novel physical concepts, such as deconfined quantum critical point [31] and algebraic spin liquid [20], can be well demonstrated within theories based on QED_3.

In 1984, Pisarski [1] studied the chiral behavior of QED_3 by solving the Dyson–Schwinger equation for fermion self-energy. He found DCSB for arbitrary fermion flavor N. Afterwards, Appelquist *et al* [3] investigated the Dyson–Schwinger equation in the lowest order of $1/N$ expansion and predicted that DCSB takes place only when the fermion flavor is less than a critical value $N_c = 32/\pi^2$. Considering the next to leading order terms [4], the critical number changes to $N_c = 128/3\pi^2$. However, the simple treatment of $1/N$ was criticized by Pennington and co-workers. They used a different truncation of the fermion DS equation and found that chiral symmetry is broken for all values of N and the generated mass is an exponential decrease for increasing N [7]. In their approach, the wavefunction renormalization was taken into account and the vertex function was chosen carefully to satisfy the Ward–Takahashi identity, multiplicative renormalizability and gauge covariance. The longitudinal part of the vertex was determined by Ball and Chiu (BC) [32], while the transverse part was proposed by Curtis and Pennington (CP) [33]. But the vacuum polarization in the gauge boson propagator contains only contribution from massless fermions. Afterwards, Maris considered the coupled DS equations of the fermion and gauge boson propagators accompanied with a set of simplified vertex functions [8]. The critical fermion flavor so obtained was found to be about $N_c = 3.3$. Based on an inequality that restricts the number of degrees of freedom in strongly coupled field theories, Appelquist *et al* [34] argued that N_c might be less than 3/2, much lower than the value estimated by the DS equation approach. However, the validity of this conjectured constraint has not been well justified. Recently, Fischer *et al* resolved a set of coupled DS equations using the vertex given by BC and CP and found that $N_c \approx 4$ [10]. Some numerical simulations on lattice QED_3 claim to find no decisive sign of DCSB for $N \geqslant 2$ [35]. The absence of DCSB in these numerical work was attributed to the large infrared cutoff in lattice studies [36] and the smallness of the generated mass scale.

In this paper, we consider DCSB in QED_3 when the gauge field also couples to scalar fields. This model can be considered as the effective theory of high-temperature superconductors. In the slave-boson treatment of the t-J model of high-temperature superconductors, the elementary excitations are the neutral, spin-$\frac{1}{2}$ spinons and the charged, spin-0 holons [24]. They both interact with an internal U(1) gauge field, which originates from enforcing the no-double occupation constraint at each site [24]. Due to the d-wave gap symmetry, the low-energy spinon excitations are massless Dirac fermions ψ. The holon can be represented by a scalar field ϕ, whose vacuum expectation value plays the role of the superconducting order parameter. Thus the effective low-energy theory of high-temperature superconductors is actually a (2+1)-dimensional QED theory of massless fermions and scalar bosons [18, 19, 24]. Physically, DCSB corresponds to the formation of antiferromagnetic long-range order [15, 19]. The gapless spin wave excitation in the antiferromagnetic ground state is the massless Goldstone boson that arises from the breaking of continuous chiral symmetry. Once the gapless fermionic spinons acquire a finite gap via the mechanism of DCSB, they are in the confinement phase [12] and hence cannot be excited at low temperatures. This fact was used by us [22] to explain the absence of residual thermal conductivity at zero temperature in underdoped cuprate superconductors.

In the effective gauge theory of high-temperature superconductors, there is no direct Yukawa coupling $\phi\bar{\psi}\psi$ between the fermion field ψ and the scalar field ϕ. However, the scalar fields have important influence on the dynamics of massless fermions by modifying the propagator of gauge field. The superconducting phase is of particular interest. As is well known, we should introduce the Anderson–Higgs mechanism when the local gauge symmetry is spontaneously broken by a nonzero vacuum expectation value $\langle\phi\rangle$. Such a mechanism generates a finite gauge boson mass m_A, which weakens the coupling strength of gauge interaction. Considering the fact that fermion vacuum condensation is mediated by a strong gauge force, it is reasonable to anticipate that a large m_A will suppress DCSB.

Introducing the scalar fields, the DS equations will become complicated for there are more propagators and vertexes. As a first step, we only consider the leading terms of $1/N$. In a previous paper [37], we considered the effect of the gauge boson mass m_A on DCSB and found that the critical number N_c is reduced by an increasing m_A. When computing the contribution of the scalar field to the Dyson–Schwinger equation, some approximations have been made. Specifically, though the gauge boson propagator contains the finite mass m_A, the Anderson–Higgs mechanism was not entirely reflected in the vacuum polarization function of the scalar field, $\Pi^B(q)$. Moreover, for simplicity, the mass of the scalar field was set to be zero, with the expectation that it has minor influence on N_c. In this paper, we give a more refined treatment on the effects of the gauge boson mass m_A and Higgs boson mass m_h on the Dyson–Schwinger equation. By detailed analytical and numerical calculations, it is shown that, for large $r = m_h/m_A$, these two parameters reduce the critical value N_c.

In section 2, we obtain the gauge boson propagator in the gauge symmetry broken phase to the one-loop level. In section 3, we study the corresponding Dyson–Schwinger equation for fermion self-energy using the bifurcation theory and parameter imbedding method. We end in section 4 with a brief conclusion.

2. Gauge boson propagator in the gauge symmetry broken phase

The Lagrangian for (2+1)-dimensional QED with N flavors of massless fermions is

$$\mathcal{L}_F = \sum_{i=1}^{N} \overline{\psi}_i \gamma^\mu (\mathrm{i}\partial_\mu - eA_\mu)\psi_i - \frac{1}{4}F_{\mu\nu}^2, \tag{1}$$

where the fermion field ψ is four-component and γ_μ are 4×4 matrices. In the context of high-temperature superconductors, the physical fermion flavor is 2, reflecting the two spin components. At present, we assume a general flavor N and use $1/N$ as the small parameter for perturbative expansion. There is an additional coupling between complex scalar fields and U(1) gauge field[1], described by

$$\mathcal{L}_B = \sum_{i=1}^{N} [|(\partial_\mu + \mathrm{i}eA_\mu)\phi_i|^2 - \mu^2|\phi_i|^2 - \lambda|\phi_i|^4]. \tag{2}$$

This model is usually called the Abelian Higgs model, or relativistic Ginzburg–Landau model. The flavor of scalar boson is also N.

We work in the Euclidean space. The gauge boson propagator $D_{\mu\nu}(q)$ is given by

$$D_{\mu\nu}^{-1}(q) = D_{\mu\nu}^{(0)-1}(q) + \Pi_{\mu\nu}(q), \tag{3}$$

where $D_{\mu\nu}^{(0)}$ is the free gauge boson propagator and $\Pi_{\mu\nu}(q)$ is the vacuum polarization tensor. Since the gauge field couples to both fermions and scalar bosons, the one-loop vacuum polarization contains two parts: $\Pi_{\mu\nu}^F(q)$ from fermions and $\Pi_{\mu\nu}^B(q)$ from bosons. The fermion part $\Pi_{\mu\nu}^F(q)$ is

$$\Pi_{\mu\nu}^F(q) = \alpha \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{\mathrm{Tr}[\gamma_\mu \not{k} \gamma_\nu (\not{q} + \not{k})]}{k^2(q+k)^2}, \tag{4}$$

[1] In realistic applications to high-temperature supreconductor, the holon field should be non-relativistic. However, the physics of holons has not been fully understood [18–20, 24]. In most of the early treatments, the holon sector was either simply neglected [18, 20] or crudely approximated [19]. In order to use the standard methods of quantum field theory, here we write a model of the standard relativistic scalar QED_3. This modification is not expected to change the basic conclusion about DCSB.

where $\alpha = Ne^2$, which is fixed as $N \to \infty$. A straightforward calculation gives the result

$$\Pi^F_{\mu\nu}(q) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{\alpha q}{8}, \tag{5}$$

where q is the momentum in three-dimensional Euclidean space.

We now deal with the scalar boson contribution $\Pi^B_{\mu\nu}(q)$. For clarity, we retain one flavor in the following and add N in the final expression of $\Pi_{\mu\nu}$. If $\mu^2 > 0$, the scalar field has a vanishing vacuum expectation value $\langle\phi\rangle = 0$. The Lagrangian is invariant under the local gauge transformation $\phi(x) \to \mathrm{e}^{\mathrm{i}\theta(x)}\phi(x)$. The one-loop polarization function has the form

$$\Pi^B(q) = \frac{e^2}{8\pi}\left[-\frac{2\mu}{q^2} + \frac{q^2+4\mu^2}{q^2 q}\arcsin\left(\frac{q^2}{q^2+4\mu^2}\right)^{1/2}\right], \tag{6}$$

with μ being the scalar boson mass. At the limit $\mu = 0$, the polarization reduces to $\Pi_B(q|) = e^2/16q$, which is the previously used expression [37].

If $\mu^2 < 0$, the scalar field ϕ acquires a finite vacuum expectation value

$$\phi_0 = \sqrt{-\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}, \tag{7}$$

with $v = \sqrt{-\mu^2/\lambda}$. The nonzero $\langle\phi\rangle$ spontaneously breaks the continuous local gauge symmetry and hence would, according to the Goldstone theorem, lead to a massless Goldstone boson. However, the Goldstone boson can be eliminated by a particular gauge transformation. Meanwhile, the originally massless gauge boson acquires a finite mass. This mechanism was proposed by Anderson in condensed matter physics and Higgs in particle physics, thus endowed with the name of the Anderson–Higgs mechanism.

It is convenient to decompose the scalar field in the following form:

$$\phi(x) = \frac{v + h(x) + \mathrm{i}\varphi(x)}{\sqrt{2}}. \tag{8}$$

Inserting this expression into the Lagrangian and expressing μ^2 by v and λ, we have

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_I,\\
\mathcal{L}_0 &= \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{4}F^2_{\mu\nu} + \frac{1}{2}e^2v^2A^2_\mu - \frac{1}{2}2\lambda v^2h^2,\\
\mathcal{L}_I &= \frac{1}{2}e^2A^2_\mu(h^2 + 2vh + \varphi^2) - e\varphi A^\mu\partial_\mu h + e(v+h)A^\mu\partial_\mu\varphi\\
&\quad - \frac{\lambda}{4}(h^4 + \varphi^4 + 4vh^3 + 4vh\varphi^2 + 2h^2\varphi^2).
\end{aligned} \tag{9}$$

It is easy to see that the mass of the Higgs boson is $m_h = \sqrt{2\lambda}v$ and the mass of the gauge boson is $m_A = ev$. Since the Landau gauge is especially useful for analyzing models of symmetry breaking, we will adopt it. In this gauge, the $evA^\mu\partial_\mu\varphi$ term disappear. The one-loop corrections to gauge boson propagator contain four diagrams, see figure 1.

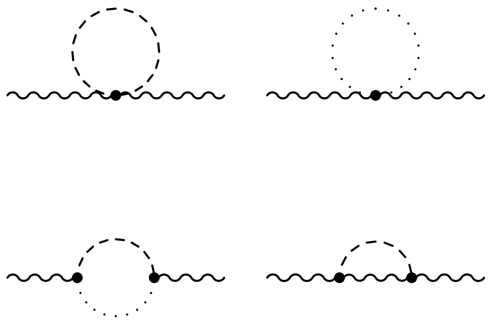

Figure 1. One-loop corrections for gauge field from scalar fields. The dashed line represents the Higgs field h, the doted line represents the massless field φ, the wiggly line represents the gauge field.

Calculating these diagrams using dimensional regularization, we obtain

$$
\begin{aligned}
\Pi^1_{\mu\nu}(q) &= -\frac{e^2 m_h}{4\pi}\delta_{\mu\nu}, \\
\Pi^2_{\mu\nu}(q) &= 0, \\
\Pi^3_{\mu\nu}(q) &= \frac{e^2}{8\pi}\delta_{\mu\nu}\left[\frac{q^2-m_h^2}{q^2}m_h + \frac{(q^2+m_h^2)^2}{2q^3}b\right] \\
&\quad + \frac{e^2}{8\pi}\frac{q_\mu q_\nu}{q^2}\left(\frac{q^2+3m_h^2}{q^2}m_h - \frac{q^4+2m_h^2q^2+3m_h^4}{2q^3}b\right), \\
\Pi^4_{\mu\nu}(q) &= \frac{e^2}{8\pi}\delta_{\mu\nu}\left[\frac{m_A^2}{q^2}m_h + \frac{q^2+m_h^2-m_A^2}{q^2}m_A - \frac{(q^2+m_h^2)^2}{2q^3}b\right. \\
&\quad \left. + \frac{(q^2+m_h^2-m_A^2)^2-4m_A^2q^2}{2q^3}a\right] \\
&-\frac{e^2}{8\pi}\frac{q_\mu q_\nu}{q^2}\left[\frac{3m_A^2}{q^2}m_h + \frac{3(q^2+m_h^2-m_A^2)}{q^2}m_A - \frac{3(q^2+m_h^2)^2}{2q^3}b\right. \\
&\quad \left. + \frac{3(q^2+m_h^2-m_A^2)^2+4m_A^2q^2}{2q^3}a\right],
\end{aligned} \tag{10}
$$

where we have used the abbreviation [38]

$$
\begin{aligned}
a &= \arctan\frac{q^2+m_A^2-m_h^2}{2m_hq} + \arctan\frac{q^2+m_h^2-m_A^2}{2m_Aq}, \\
b &= \arctan\frac{q^2-m_h^2}{2m_hq} + \frac{\pi}{2}.
\end{aligned} \tag{11}
$$

Adding all these contributions yields the total one-loop polarization tensor $\Pi_{\mu\nu}$

$$
\begin{aligned}
\Pi_{\mu\nu}(q) &= \Pi^F_{\mu\nu}(q) + \Pi^B_{\mu\nu}(q) \equiv \Pi_1(q)\delta_{\mu\nu} - \Pi_2(q)\frac{q_\mu q_\nu}{q^2} \\
\Pi_1(q) &= \frac{\alpha q}{8} + \frac{\alpha}{8\pi}\left[-m_h + m_A + \frac{m_h^2-m_A^2}{q^2}m_A + \frac{m_A^2-m_h^2}{q^2}m_h\right. \\
&\quad \left. + \frac{(q^2+m_h^2-m_A^2)^2-4m_A^2q^2}{2q^3}a\right]
\end{aligned}
$$

$$\Pi_2(q) = \frac{\alpha q}{8} + \frac{\alpha}{8\pi}\left[-m_h + 3m_A + 3\frac{m_h^2 - m_A^2}{q^2}m_A + 3\frac{m_A^2 - m_h^2}{q^2}m_h - \frac{q^2 + 2m_h^2}{2q}b + \frac{3(q^2 + m_h^2 - m_A^2)^2 + 4m_A^2 q^2}{2q^3}a\right]. \quad (12)$$

Note that, in the symmetry broken phase, the gauge boson is massive and the vacuum polarization is not transverse. (This fact can be understood from the general Ward–Takahashi identity of the Abelian Higgs model [39].) The full gauge boson propagator is

$$\begin{aligned} D_{\mu\nu}(q) &= D^{(0)}_{\mu\nu}(q) - D^{(0)}_{\mu\rho}(q)\Pi^{\rho\sigma}(q)D^{(0)}_{\sigma\nu}(q) + \cdots \\ &= \frac{1}{q^2 + m_A^2 + \Pi_1(q)}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right). \end{aligned} \quad (13)$$

3. Dynamical chiral symmetry breaking

We are now ready to study DCSB. The Dyson–Schwinger equation for the fermion propagator is given by

$$S_F^{-1}(p) = S_F^{(0)-1}(p) + e^2\int\frac{d^3k}{(2\pi)^3}\gamma^\mu S_F(k)D_{\mu\nu}(p-k)\Gamma^\nu(k,p). \quad (14)$$

To the leading order in $1/N$ expansion, the vertex function Γ^ν is replaced by the bare vertex γ^ν. In the Landau gauge, the inverse fermion propagator can be written as

$$S_F(p)^{-1} = i\not{p} + \Sigma(p), \quad (15)$$

where the wavefunction renormalization is neglected. Taking trace on both sides of the DS equation, we arrive at a closed integral equation for fermion self-energy

$$\Sigma(p) = \frac{e^2}{4}\int\frac{d^3k}{(2\pi)^3}\frac{\Sigma(k)}{k^2 + \Sigma^2(k)}\mathrm{Tr}[\gamma^\mu D_{\mu\nu}(p-k)\gamma^\nu]. \quad (16)$$

Inserting the gauge boson propagator (13) into this equation leads to

$$\Sigma(p) = \frac{2\alpha}{N}\int\frac{d^3k}{(2\pi)^3}\frac{\Sigma(k)}{k^2 + \Sigma^2(k)}\frac{1}{(p-k)^2 + m_A^2 + \Pi_1(p-k)}. \quad (17)$$

If this equation develops a nontrivial solution, then the massless fermion acquires a finite mass, which signals the occurrence of DCSB.

Before performing numerical computations, we would like to make some qualitative analysis. Note that the Higgs mass $m_h = \sqrt{2\lambda}v$ and the gauge boson mass $m_A = ev$ are not independent quantities. We can choose either as the turning variable and study how it affects the critical flavor N_c. Their ratio

$$r = \frac{m_h}{m_A} = \frac{\sqrt{2\lambda}}{e} \quad (18)$$

is considered as an external parameters.

When m_A (and hence m_h) approaches zero, the polarization $\Pi_1(q)$ becomes

$$\frac{\alpha q}{8} + \frac{\alpha q}{16}. \quad (19)$$

The first term is the contribution from the fermion loops, while the $\alpha q/16$ term reflects the additional coupling between the massless gauge field and the scalar fields. In this limit, following Appelquist [3], the integral equation can be converted to a differential equation giving the critical flavor $N_c = 64/3\pi^2$.

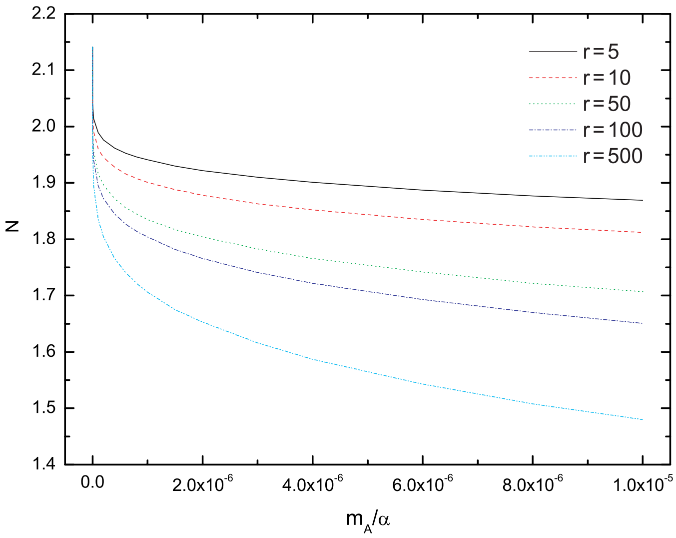

Figure 2. The variation of N_c from m_A for different ratio $r = m_h/m_A$.

Since the main contribution in the integral comes from the momenta $|p-k| < \alpha$, we can expand the denominator in terms of $|p-k| = q$. The function a can be written as

$$a = 2\left[\frac{q}{m_h + m_A} - \frac{q^3}{3(m_h+m_A)^3} + \frac{q^5}{5(m_h+m_A)^5} - \cdots\right]. \tag{20}$$

Then the leading terms of the denominator in the integral is

$$m_A^2 + \frac{\alpha q}{8} + \frac{\alpha m_A}{12\pi}\frac{r^2+r-8}{r+1}. \tag{21}$$

For relatively large r, it is an increasing function of both m_A and r variables. So one can guess that the critical flavor N_c is a decreasing function of both m_A and r.

When $q = 0$, the polarization $\Pi_1(0)$ becomes negative for small r, thus the denominator in the integral may become negative. However, for the system considered in our paper, this situation can never be met. Here, the ratio r is the Ginzburg–Landau parameter κ ($r = \sqrt{2}\kappa$) [38]. A large value of r corresponds to the type-II regime. The high-temperature superconductors are all extreme type-II superconductors and the Ginzburg–Landau parameter is generally about 100. Thus we only consider the relatively large value of r.

We apply the bifurcation theory and parameter imbedding method to solve this nonlinear integral equation. The basic idea and detailed computation procedures are presented in previous papers [37, 40]. To determine the bifurcation point, which separates the chiral symmetric phase and chiral symmetry broken phase, we only need to find the eigenvalues of the associated linearized equation. Taking the Frêchet derivative of the nonlinear integral equation, we obtain

$$\Sigma(p) = \frac{2\alpha}{N}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{k^2}\frac{\Sigma(k)}{(p-k)^2 + m_A^2 + \Pi_1(p-k)}. \tag{22}$$

In the numerical calculations, the mass terms (m_h, m_A and Σ) and momenta (p, k) are scaled by α. The whole results are presented in figure 2. From the figure, we see that: the critical flavor N_c really decreases as the turning variable m_A/α and the ratio r increases. As the mass tends to zero, the corresponding value of N_c is about 2.14, just about the expected value $64/3\pi^2$.

DCSB is essentially a low-energy phenomenon and takes place only in field theories that are asymptotic free. To trigger fermion–anti-fermion condensation, the gauge force must be strong enough at low energy (or, large distance). In the gauge symmetry broken phase, as the gauge boson mass m_A becomes larger, the attraction force between fermion and anti-fermion is weaker. For a fixed physical flavor N, DCSB is completely suppressed when m_A exceeds some critical value. The polarization function Π_1 also affects the critical flavor N_c. For large r, the one-loop correction $\Pi_1(q)$ increases as m_A becomes larger, making the critical flavor N_c to further decrease.

4. Conclusion and discussion

We studied the DS equation for fermion self-energy in QED_3 with an Abelian Higgs model at the leading order of $1/N$. In the gauge symmetry broken phase, the gauge boson mass and the Higgs boson mass both alter the critical number for fermion to acquire dynamically generated mass. By numerically solving the DS equation, it is found that, for large r, the critical flavor N_c is a monotonously decreasing function of the gauge boson mass m_A, and also of the Higgs boson mass m_h.

In realistic applications to high-temperature superconductors, the parameter r always has a large value. However, from a purely theoretical point of view, the region of small r is rather interesting. As parameter r decreases, the system would undergo a type-II to type-I transition at some particular value r_c . The impact of such transition on DCSB is unknown and deserves further exploration.

One important issue is the validity of the approximations used in our treatment of the DS equation. In the present work, the approximation is the same as that adopted by Appelquist *et al* [3]. The qualitative prediction of the finiteness of critical flavor N_c and the crude value of N_c obtained under this approximation have been confirmed by most (though not all) of the subsequent investigations. To go beyond the present approximation, we need to include the self-consistent equations of wavefunction renormalization and gauge boson polarization, and to consider various vertex corrections, as in the recent publication of Fischer *et al* [10]. Due to the complexity of vacuum polarization brought by the Higgs mechanism, we would like to consider these corrections in the forthcoming work.

Acknowledgments

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